Impedance Measurement

rev. 1

Purpose: Impedance measurement using an LCR Meter and with a custom-made digital Ohmeter.

Summary of theory

In sinusoidal steady state regime, the impedance, $Z = \frac{\underline{U}}{\underline{I}}$, and the admitance, $Y = \frac{\underline{I}}{\underline{U}} = \frac{1}{Z}$, are defined. $\underline{U}$ and $\underline{I}$ represent the phasor of the voltage, and the one of the intensity of the electrical current, respectively, from Fig. 1a.

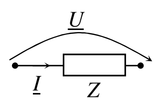


Figure 1a

These measures are complex, and they can be written in algebraic form as follows:

$$Z = R + jX, \qquad Y = G + jB$$

R – series resistence
X – series reactance (*X* >0 for inductive impedances and *X* <0 for capacitive impedances)
G – parallel conductance
B – parallel susceptance (*B* <0 for inductive admittances and *B* >0 for capacitive admittances)

The relation between themagnitude of the impedance and the admittance is easily obtained:

$$R = \frac{G}{G^2 + B^2} \qquad X = -\frac{B}{G^2 + B^2}$$

In exponential form, the admittance and the impedance can be written as follows:

$$Z = |Z| \cdot e^{j\phi_Z} \text{ respectively } Y = |Y| \cdot e^{j\phi_Y}$$

where $\phi_Z = \phi_U - \phi_I = -\phi_Y$ and $|Z| = \frac{|U|}{|I|} = \frac{1}{|Y|}$

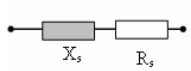
The model of a reactance with losses

A reactance with losses, having a quality factor Q at frequency f, is considered. There are two possible models for this circuit: series and parallel. They are depicted in Figure 1b.

X can be the reactance of an inductor, or the reactance of a capacitor, respectively.

$$X_L = \omega L$$

$$X_C = -\frac{1}{\omega C}$$



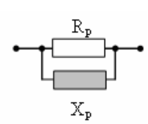


Figure 1b

For the two models the quality factors Q_s and Q_p are defined:

$$Q_s = \frac{|X_s|}{R_s} = \frac{\omega L_s}{R_s} = \frac{1}{\omega R_s C_s}$$

$$Qp = \frac{R_p}{|X_p|} = \frac{R_p}{\omega L_p} = \omega R_p C_p$$

As the two quality factors are defined for the same physical reactance, they should have equal values.

$$Q_s = Q_p = Q$$

For a reactance with losses, the loss factor, D, is defined:

$$D = \frac{1}{Q}$$

The relations between the elements of the two models, at a fixed frequency f, are :

$$X_p = X_s\left(1 + \frac{1}{Q^2}\right) = X_s\left(1 + D^2\right)$$

$$R_p = R_s\left(1 + Q^2\right)$$

The equivalence relation between the reactances can also be written as follows, depending on the nature of the reactance, capacitive or inductive :

$$L_p = L_s\left(1 + 1/Q^2\right)$$
$$C_s = C_p\left(1 + 1/Q^2\right) \qquad (1)$$

Frequency-dependent behavior of a series LC circuit

The impedance of a LC series circuit strongly depends on the frequency, and it is equal to :

$$Z(\omega) = j\omega L + \frac{1}{j\omega C}$$

If the impedance is predominantly capacitive, then it can be written as follows:

$$Z(\omega) = \frac{1}{j\omega C}\left(1 - \omega^2 LC\right) = \frac{1}{j\omega \frac{C}{1 - \omega^2 LC}} = \frac{1}{j\omega C_e}$$

where

$$C_e = \frac{C}{1 - \omega^2 LC} \qquad (2)$$

The equivalent capacitance varies with the frequency. Moreover, after the frequency increases above the resonance frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} \qquad (3)$$

the capacitance changes sign (after the resonant frequency inductive effect prevails), where

$$L_e = L\left(1 - \frac{1}{\omega^2 LC}\right) \qquad (4)$$

Frequency-dependent behavior of a parallel LC circuit

The admittance of a parallel LC circuit is

$$Y(\omega) = j\omega C + \frac{1}{j\omega L}$$

If the admittance is predominantly inductive, then it can be written as follows:

$$Y(\omega)=\frac{1}{j\omega L}\left(1-\omega^2 LC\right)=\frac{1}{j\omega\frac{L}{1-\omega^2 LC}}=\frac{1}{j\omega L_e}$$

where

$$L_e=\frac{L}{1-\omega^2 LC}$$

The equivalent inductance varies with the frequency. Moreover, after the frequency increases above the resonance frequency, $\omega_r=\frac{1}{\sqrt{LC}}$, the inductance changes sign (after the resonant frequency capacitive effect prevails), where

$$C_e=C\left(1-\frac{1}{\omega^2 LC}\right)$$

Remark: The two models, previously presented, can also be used for the frequency-dependent behavior of the reactive element (L,C) because of the parasitic reactance (C_{pz}, L_{pz}) and, when, generally speaking, the reactive element is not constant with the frequency.

The quadripolar measurement configuration

When the measured impedance has small values, or when measuring probes have significant length (distance measurement), the impedance of the probes and that of the contact resistences may not be negligible, being comparable with the measured impedance, Z_x. The measuring principle uses two terminals at each lead of the impedance. A pair of terminals is used to inject current through the unknown impedance Z_x, the other to measure the voltage that drops on Z_x. The configuration is named *quadripolar* because of the 4 terminals. The two pairs of terminals are connected as close as possible to the body of the impedance.

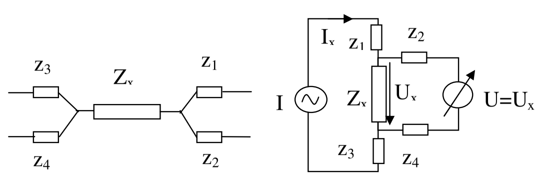


Figure 2a: The Quadripolar Model

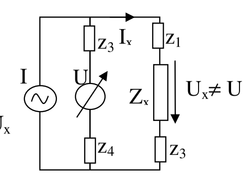


Figure 2b: The Bipolar Model

The four unwanted impedances (of the 4 terminals) are z_1, z_2, z_3, z_4. As z_2 and z_4 are in series with the voltmeter, which has very high input impedance, they are negligible. Because z_1 and z_3 are in series with the current source, which has high internal impedance, they are also negligible. Therefore, this configuration allows minimizing the undesirable effect of the 4 impedances, making them to appear in series with other higher impedances that already exist in the circuit. If only two terminals are used (Figure 2b, bipolar configuration), the "current" and "voltage" paths can no longer be separated, and the impedance including also the impedance of the probes, is measured :

$$Z_m=Z_x+z_1+z_3$$

which is a systematic error :

$$\varepsilon_{Z_x}{}^s=\frac{z_1+z_3}{Z_x}$$

For example, when measuring a resistance R, using connecting wires which have the resistance r, in the bipolar configuration (two terminals only), a systematic error is obtained :

$$\varepsilon_R{}^s=\frac{2r}{R}$$

Ohmmeter with Operational Amplifier

Ohmmeters made only with passive components and a power source (battery), as the ones in analog multimeters, have the disadvantage of a non-linear scale: given a voltage source of value E, the current I=E/R_x, therefore the current through the meter is in inverse proportion to the resistance. This is not a serious problem for analog meters (with a pointer moving in front of a scale), because an additional scale, according to the law 1/R, can be drawn. But for digital multimeters, a linear scale is essential, because, the digital voltmeter from the multimeter is linear, and any measured value vhich must be converted to a voltage must follow a linear relation.

A solution that uses active components is given in the schematic from Figure 3

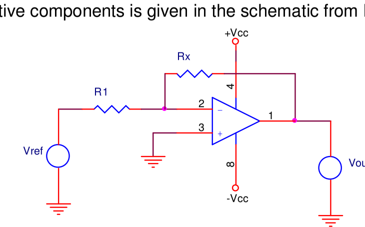


Figure3: Ohmmeter with linear scale, theoretical version

Because of the inverter configuration, the conversion relation can be written as follows :

V_{out}= - R_x/R_1 V_{Ref}

equivalent to a direct proportionality relation :

V_{out}= - K V_{Ref}

__Measurements__

1. ***Resistance measurement using the LCR - meter***

The LCR-meter is a device that allows the automatic measurement of 2 chosen parameters of an impedance (selectable from the **MODE** knob).

Measure three available resistances, with different values, using the LCR-meter, with the following settings: **SPEED->MEDI**, **DISPLAY -> VALUE**, **MODE -> R/Q**, **CIRCUIT -> SERIES**. These settings can be adjusted by pressing the knobs from the right of the display. The working frequency is implicitly set on 1kHz (verify on the display, and, if the frequency has not that value, press the softkey **FREQ** - the same as the key "-", type the chosen frequency, and press the softkey **ENTER**). Determine the absolute, and the relative, errors between the measured value and the value marked on the resistance, in each case.

2. ***Using the sort method for determining the tolerance of a resistance***

To configure the sort mode enter the **MENU** mode, and select the **SORT** mode. In the **SORT** mode, define the nominal value of the resistances to be sorted: select **NOM. VAL**, and insert, from the keyboard, the nominal value of the resistance, for which the tolerance is to be determined. After keying in the value, press the **ENTER** key. Press **EXIT** to exit the menu. Select the display mode **DISPLAY -> DELTA**, and read the displayed value (the difference between the displayed value and the nominal one), then select the mode **DISPLAY -> DELTA%**, and read the displayed value (the tolerance in percentage).

In this operating mode, measure the tolerance of the three resistances (the percent value).

Remark: This approach can be used to sort a set of resistances: select the sorting mode, define the nominal value and then select the resistances for which the tolerance in percentage (displayed in **DELTA%** mode) is less than the chosen value.

3. *Measurement of small resistances*

Use and compare the bipolar and the quadripolar configuration, when measuring a very small value resistor (the smallest value available, units of Ω).

a) Connect the unknown value resistor at the probes of the LCR-meter. Observe that the adapter connected to the LCR-meter uses 4 plugs, the measurement being quadripolar, each of the two crocodile clips connecting 2 plugs at one terminal of the unknown impedance, as in figure 2a. The settings are the ones from point 1 (return to the display mode **-> VALUE**). Write down the value indicated for the resistance.

b) Remove the adapter. (***Be careful :*** Please, handle the adapter carefully. In order to remove it, rotate the brown plastic levers to the left, looking to obtain the alignment of the BNC jacks which are attached to them, so that the adapter can be easily removed - **DO NOT FORCE THEM !** *When reconnecting the adapter, after inserting the BNC jacks, rotate the levers to the right, to fix them*). Connect to the plugs **L**FORCE, **H**FORCE, a regular cable with crocodile clips, to each one. Connect the black crocodiles (the ground wires) together, and connect the red crocodiles to the resistance to be measured. Write down the indicated value. Why is there a difference between the two measured values (at ***a*** and ***b***) ?

c) Remove the resistor and connect together the two red crocodiles. Write down the indicated value, which will not be equal to zero. What does this value represent ? How much is the absolute systematic error (ΔR) done when measuring the resistance ?

d) Determine the value of the resistor correcting the systematic error (substract the value of the crocodile cables, from ***c***, from the value measured at ***b***). Determine the relative error of this value comparing to the value determined at ***a.***

Why is that, at ***a***, the determination of the resistance of the cables was not necessary ?

e) Measuring the resitance of a wire: replace the resistor with one of the wires from the electronic multimeter, and measure it, in the bipolar configuration. Reconnect (***CAREFULLY***) the *measurement adapter*, and measure the wire in quadripolar configuration. Calculate the relative error of the bipolar configuration comparing to the quadripolar one.

4. *Measurement of capacitors and coils*

a) *measurement of capacitors*

Measure two available capacitors. Select the mode **MODE -> C/D**, series model (**CIRCUIT->SERIES**), and write down the values C_s and D. Select the parallel model (**CIRCUIT -> PARALL**) and write down the value C_p. The value of D is the same. Calculate $Q = \frac{1}{D}$. How are, generally speaking, the values of the quality factors of the capacitors ?

How are the C_s and C_p values ? Why ?

b) *measurement of inductors*

Measure the inductor, existing on the table (*Attention ! Do not look for core inductors. The inductors available in the laboratory look like green resistors marked in the color code).* Select the mode **MODE -> L/Q**, and measure for the inductor, the series model (L_s and Q), (**CIRCUIT->SERIES**), and the parallel model (L_p and Q), (**CIRCUIT -> PARALLEL**). Determine the value of the resistance R_s from the definition of the quality factor. Calculate Q_{calc}, using the relation between the series and the parallel model (relation 1). Compare Q with Q_{calc}.

How are, generally speaking, the *usual* Q values of inductors, comparing to the ones for capacitors (do not refer to special cases) ?

Measure the inductor (**MODE -> L/Q, CIRCUIT->SERIES)**, at the frequencies $f_2 = 10kHz$, $f_3 = 33kHz$, $f_4 = 66kHz$, $f_5 = 100kHz$. Modify the frequency pressing the key **FREQ**. Type the chosen value in kHz, and press the **ENTER** key. What happens with the value of the inductor ? Explain the obtained results.

5. *Measuring the RC circuit*

Measure a RC circuit series, made on the *breadboard*. Use the capacitor with high value (≥ 47nF) and the resistance of tens of de Ω. For the resulted circuit, measure C_s, C_p and D. Calculate Q, using the value measured for D. Calculate Q_{calc} using the relation between the series and the parallel model (relation 1). Compare Q_{calc} with Q. Proceed as at 4, when measuring capacitors. The working frequency is 1kHz (the implicit value).

Determine the resistance of the RC circuit: select the display mode **MODE -> C/R** and determine the value of the resistance for the series model (R_s) and for the parallel model (R_p).

Calculate the value of the resistance R_s from the loss factor D.

Modify the frequency to 100kHz. Measure the values C_s, D. What happens with the value of the capacitor, C_s? Connect the measurement probes at the terminals of the resistance, select the mode **MODE -> L/Q** and write down the value indicated for L (the frequency is 100kHz). How can the variation of the value measured for the capacitor, be explained ?

6. *Measuring the frequency-dependent behavior of a LC circuit*

Measure the frequency-dependent behavior of a LC series circuit, made on the *breadboard*. Use the available inductor, and the capacitor with high value (≥ 47nF). First, separately measure the 2 components, L,C using the LCR-meter at the frequency of 1KHz, then measure the (equivalent) inductance and the (equivalent) capacitance of the LC *series* circuit at the frequencies : $f_1=1kHz$ $f_2=5kHz$, $f_3=10kHz$, $f_4=12kHz$, $f_5=15kHz$, $f_6=20kHz$, $f_7=50kHz$, $f_8=100kHz$. Measure using the modes **MODE -> L/Q**, **CIRCUIT->SERIES**, **MODE -> C/D**, **CIRCUIT->SERIES**, respectively.

Calculate the theoretical value of L_e or C_e (equivalent) of the circuit, for the mentioned frequencies, according to relations (2) and (4), where *L* and *C* are the ones determined at low frequency (1KHz).

Calculate the resonance frequency (relation 3). What happens with the nature (inductive or capacitive) of the equivalent impedance of the circuit, when the measurement frequency is higher than the resonance frequency ?

7. *The study of a digital ohmmeter with linear scale*

The schematic from figure 3 is based on the inverter "classical" configuration of an Operational Amplifier, powered by a differential supply (+/- Vcc from the ground), namely a symmetrical double supply. The existence of a negative supply allows a negative voltage at the output, when the input is positive. But for portable devices, powered by a battery, the existence of a double supply is uneconomical. A possible solution is using a single supply having the value Vcc, and creating a local ground, a "conventional" ground, by means of a resistive divider. On the schematic from figure 4, the conventional ground, having the symbol is at Vcc/2 below the (+) plug of the power supply, and with Vcc/2 above the ground plug of the power source. The conventional ground will be the reference ground of the input and the output signals. Consequently, the maximum excursion of these signals will be +/- Vcc/2 around this ground. Now there is possible to obtainin positive and negative voltages, referenced to the conventional ground .

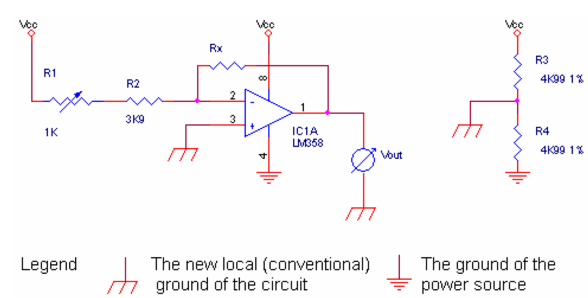


Figure 4: Linear - Scale Ohmmeter, practical version

In order not to complicate the circuit with an additional voltage, use Vcc/2 from the conventional ground (meaning Vcc from the ground), as reference voltage, V_{ref} from figure 3 (the input voltage), namely the same voltage as the pin 8 of the Op-Amp. The output voltage (observe that the voltmeter is connected between the output and the ground) is:

$$V_{out} = -\frac{V_{CC}}{2}\frac{R_X}{R_1 + R_2} = -K\,R_X$$

For ease of reading of the digital voltmeter, *K=1V/KΩ* is imposed. As the voltage supplied by the voltage supply from the laboratory is not very precise (approximately 8...10 V), use the adjustable potentiometer R_1 to finely vary its resistance, in order to obtain the value 1 for *K*.

Remark 1. The output voltage can not have values over +/- $V_{cc}/2$ referenced to because of the Op-Amp. If *K=1* and V_{cc}=9V, this circuit can not measure resistances which have values higher than 4.5 KΩ ($R_{x\ Full\ Scale}$ = 4.5KΩ). Depending on V_{cc} from each table, this value will vary accordingly.

Remark 2. The limit obtained from the remark 1 is theoretical. In practice, no Op.-Amp. has a linear excursion between the extreme values of its supply voltages (- and +). For some Op-Amps (eg "classic" 741) there is a difference of a few V between the maximum voltage which can be obtained at the output voltage and the supply voltage. Therefore, if a 741 was used at 9V, the minimum output voltage would be about 3 .. 4V, and the maximum voltage would be about 9-3 ..4 = 5 .. 6V, so only about +/- 1V around the middle, which is the conventional ground ! These circuits, of older generations, are explicitly designed to operate at relatively high voltage double supplies (+/- 15V). The LM358 circuit was chosen because it is optimized to operate from single supplies (or *single rail*), namely the excursion of the output voltage, mentioned in the datasheet is between 0V and approximately Vcc-1.5V. There are Op-Amps that can achieve at their output voltages equal or close enough of both supply voltages (*rail-to-rail swing*).

a) Calibrating the ohmmeter*:*

Assemble, on the bread board, the schematic from figure 4. Use one of the 2 Op-Amps available in the capsule of the LM358 circuit. The pinout is given in Figure 5. Connect the adjusting resistance using the pin in the middle (the cursor) and any of the other two pins. To create the ground use any two resistances with equal values and the precision of 1%, available (not necessarily 4K99).

Pay attention ! *Verify, using the voltmeter, the value and the polarity of the supply (set on 9V) before supplying the circuit. Also verify that the* ***black plug*** *of the bread board (the ground of the power supply)is connected at the supply, the resistance R4, and pin 8 of the Op-Amp, only. Do not connect the conventional ground to that plug, also ! The conventional ground is at Vcc/2 from the "real" ground (the black plug), so it is ground only with respect to the output. Choose another group of breadboard holes, except the line used for the "real" ground , to connect the conventional ground .*

Pay attention ! *In order to measure the output voltage, connect the ground of the voltmeter (the black crocodile clip) to the conventional ground , according to the schematic ! Remove the connection of the voltmeter to the ground of the power supply !*

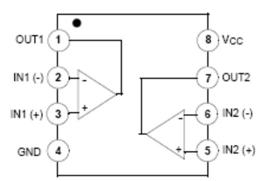


Figure 5: Integrated Circuit LM358

For the calibration, replace R_x with a precision resistance of 1 or 2KΩ (the available). Rotate the adjustable resistance until the voltage indicated on the voltmeter is -1.00V (or -2.00V).

b) Remove the precision resistance and replace it by two resistances having values lower than 3.3KΩ. Reading the display of the voltmeter, determine the values of the 2 resistances, $R_{x1,2\ measured}$.

Remove the resistances and the voltmeter from the circuit, and measure $R_{x1,2real}$, using the digital multimeter on the Ohmmeter mode. ***Pay attention !*** *do not measure using the Ohmmeter of the multimeter any resistance in the circuit ! remove the resistance from the circuit, to measure it.* Calculate the relative errors.

c) Measure, using the two methods, a resistance with higher value than 5KΩ; which is the value indicated by the ohmmeter on the bread board ($R_{x3\ measured}$) comparing to the real value ?

To explain the obtained error, measure, using the ohmmeter from the digital multimeter, the value set for the potentiometer, for calibration, as well as R_2 (removed from the circuit). Measure, also, the supply voltage. Using these values, calculate $R_{x\ Full\ scale}$ (the maximum resistance that the ohmmeter on the bread board can measure). Consider the conversion relation. How is $R_{x\ Full\ scale}$ comparing to $R_{x3\ real}$?

d) Identify and explain the error sources of that circuit.

Preparatory questions

1. For an inductor L_p=400mH and Q=50 are measured, at the frequency f=1kHz. Determine the resistance R_p and the value of the inductor for the series circuit, L_s.
2. Calculate the quality factor for a RC series circuit, having C_s=10nF and R_s=50Ω, at the frequency 1kHz.
3. Calculate the quality factor for a RC parallel circuit, having C_p=10nF and R_s=1MΩ, at the frequency 1kHz.
4. Given a LC circuit, with L=1mH and C=100/4π^2 nF. Calculate the resonance frequency of the circuit.
5. Given a LC circuit, with L=10mH and C=1/4π^2 nF. Calculate the impedance of the circuit, at the frequency 1kHz.
6. For an inductor L_s=10mH and Q=10 are measured, at the frequency f=1kHz. Determine the resistance R_s and the value of the inductor for the parallel circuit, L_p.
7. For a capacitor C_s=200nF and Q=1000 are measured, at the frequency f=10kHz. Determine the resistance R_s and the tangent loss, $D=tg\,\delta$.
8. A resistance is measured using the bipolar configuration (two terminals only). The value of the resistance is R=50Ω. The resistance of the wires is 0,5Ω. Determine the systematic error done, when measuring the resistance.
9. For an inductive impedance L_p=202mH and L_s=200mH are measured. Determine the quality factor of the impedance.
10. Given a LC circuit, with L=10mH and C=1nF, calculate the resonance frequency of the circuit.
11. For the circuit in figure 3, determine the relation between the resistance R_x and the output voltage.
12. For the circuit in figure 3, determine the measurement range for the resistance R_x, if the power supply voltage is +/-V_{cc}=+/-5V, the resistance R_1=10kΩ and the voltmeter has U_{FS}=10V.
13. Determine the error done by the voltmeter in figure 3, when measuring a resistance Rx=500Ω, if the power supply is U_{ref}=5V±1%, the resistance R_1=5kΩ±1%, and the voltmeter has U_{FS}=10V and precision class C=0,5%.
14. A LC circuit has L=1mH and C=1nF. Determine the resonance frequency of the circuit. Which is the nature of the impedance indicated by the LCR-meter for a frequency higher than the resonance frequency ?
15. An inductance has L=1mH and the parasite capacitance C_p=30pF. Which is the nature of the impedance indicated by the LCR-meter at the frequency f=100kHz?
16. For the circuit in figure 3, E=10V, R=5kΩ. The voltmeter has U_{FS} =3V. The indication of the voltmeter is U=1V. Determine R_x and R_{xFS} - the full scale resistance.